MOTION ESTIMATION OF 2D ATMOSPHERIC LAYERS WITH VARIATIONAL ASSIMILATION TECHNIQUES

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Abstract

In this paper, we aim at facing the problem of estimating time consistent mesoscale dynamics of atmospheric layers from satellite image sequences.
Due to the sudden variations of the luminance function observed within clouds patterns and which are caused by the intrinsic sparse 3-dimensional nature of clouds, the estimation of accurate dense motion field is an intricate issue.

Relying on a physically sound vertical decomposition of the atmosphere into layers, a dense motion estimator dedicated to the extraction of multi-layer horizontal wind fields is firstly described in order to provide velocity measurements.
This estimator is expressed as the minimization of a global function including a data term and a spatio-temporal smoothness term. A robust data term relying on shallow-water mass conservation model is proposed to fit sparse observations related to each layer.
These constraints are combined with a robust second-order regularizer preserving divergent and vorticity structures of the flow.

However, the produced velocity fields are successive estimations from frame to frame which suffer from temporal consistency.
Hence, we introduce a variational framework derived from data assimilation principles in order to realize a temporal Bayesian smoothing of the estimated velocity fields.

Thus, the noisy measurements are smoothed according to the Large Eddy Simulation of a vorticity-divergence based shallow-water model on each cloud layer. Following optimal control recipes, the associated minimization is conducted through an iterative process involving a forward integration of our dynamical model followed by a backward integration of an adjoint evolution law. Both evolution laws are implemented with upwind second order non-oscillatory scheme. The approach is applied on two real world meteorological satellite image sequences.

1. INTRODUCTION:

The analysis and control of complex fluid flows is a major scientific issue. In that prospect, flow visualization and extraction of accurate kinetic or dynamical measurements are of the utmost importance. For several years, the study of dynamic structures and the estimation of dense velocity fields from fluid image sequences have received great attention from the computer vision community [Corpetti02, Fitzpatrick85, Ford94, Larsen98, Memin99c, Zhou00].
Application domains range from experimental visualization in fluid mechanical to geophysical flow analysis in environmental sciences. In particular, accurate measurement of atmospheric flow dynamics is of the greatest importance for weather forecasting, climate prediction or analysis, etc...

The analysis of motion in such sequences is particularly challenging due to abrupt and sudden changes of the luminance function in image sequences. For these reasons, motion analysis techniques designed for computer vision application and quasi-rigid motions, are not well adapted in this context. Recently, methods for fluid-dedicated dense estimation have been proposed to characterize fluid motion [Corpetti02, Corpetti03, Cuzol05, Kohlberger03, Yuan06, Ruhnau07].

However, these motion estimators are still using only a small set of images and thus may suffer from a temporal inconsistency from frame to frame. The set of motion fields provided may not respect fluid mechanics conservation laws. The design of appropriate methods enabling to take into account the underlying physics of the observed flow constitutes a widely open domain of research. We are here interested in using the *vorticity-divergence* formulation of Shallow-*Water* equations which describes accurately the evolution of the flow for the filtering of noisy motion fields.

The approach we propose in this work is related to variational data assimilation principles used in meteorology
[Bennet92, Ledimet86, Talagrand87]. Such techniques enable, in the same spirit as a Kalman filter, a temporal smoothing along the whole image sequence. As does a Bayesian smoother, it combines a dynamical evolution law of state variables representing the target of interest with the whole set of available noisy measurements related to this target.
Nevertheless, unlike Kalman filtering and stochastic Bayesian filtering approaches such as particle filtering, variational assimilation techniques allows to cope with state spaces of very large dimension.

The technique we devise allows us to incorporate in the whole set of motion fields a dynamical consistency along the image sequence. The approach is expressed as the minimization of a global spatio-temporal functional. The optimization process is led through the introduction of an adjoint evolution model. This method has the advantage to provide an efficient numerical approximation of the gradient functional without resorting to the complete analytical expressions of Euler-Lagrange equations. This is particularly interesting when dealing with high order differential operators.

2. DATA ASSIMILATION PRINCIPLE:

Data Assimilation is a technique related to optimal control theory which allows estimating over time the state of a system of variables of interest. This method enables a smoothing of the unknown variables according to an initial state of the system, a dynamic law and noisy measurements of the system's state.
Let the vector of variables X represents the state of the system. The evolution of the system is assumed to be described through a (possibly non linear) differential dynamical model *M*:

$$\begin{aligned} \partial_t X + M(X) &= 0 \\ X(t_0) &= X_0 \end{aligned}$$

This system is monitored by a control variable X_0. We then assume that observations Y are available. These observations may live in a different space (a reduced space for instance) from the state variable. We will nevertheless assume that there exists a differential operator *H* that goes from the variable space to the observation space. A least squares estimation of the control variable regarding the whole sequence of measurements available within a considered time range comes to minimize with respect to the control variable X_0, a cost function of the following form:

$$J(X_0)=\int_t \|Y-H(X(X_0))\|^2 \, dt.$$

A first approach consists in computing the functional gradient through finite differences. Denoting N the dimension of the control parameter X_0, such a computation is impractical for control space of large dimension since it requires N integrations of the evolution model for each required value of the gradient functional.

Adjoint models as introduced first in meteorology by Le Dimet and Talagrand in [Ledimet86] authorize the computation of the gradient functional in a single backward integration of an adjoint variable. The value of this adjoint variable at the initial time provides the value of the gradient at the desired point. This approach is widely used in environmental sciences for the analysis of geophysical flows [Ledimet86, Talagrand87].

In this paper, we aim at applying this process to motion estimation of atmospheric layers. To that end, we now present the observations and the dynamical model used in this work.

3. VELOCITY OBSERVATIONS:

In this section, the process used to compute dense observation of motion field from satellite images is described. Since there is a loss of information induced by projection in an image plane, several hypotheses are necessary to tackle the reconstruction problem. The troposphere is the lower part of the atmosphere ranging from the sea level up to a bound called tropopause. The layering of atmospheric flow in the troposphere is valid in the limit of horizontal scales much greater than the vertical scale height, thus roughly for horizontal scales greater than 100 km. It is thus impossible to guarantee to truly characterize a layered atmosphere with a local analysis performed in the vicinity of a pixel characterizing a kilometre order scale.

Nevertheless, one can still decompose the three-dimensional space into elements of variable thickness, where only sufficiently thin regions of such elements may really correspond to common layers. Analysis based on such decomposition presents the main advantage of operating at different atmospheric pressure ranges and avoids the mix of heterogeneous observations. The analysis of such elements will either be significant of layer dynamics, or reveal an average motion significant of thick regions. In the present paper, such elements will be defined and called with abusive language ``layers''.

Let us present the three-dimensional space decomposition that we chose for the definition of the layers. The k-th layer corresponds to the volume lying in between an upper surface z^k_t and a lower surface z^k_b. In areas where clouds have their top belonging to a given pressure interval, the upper surface corresponds to the height of top of clouds.

Relatively to the different layers, transmittances are sparsely observed only in the presence of clouds in satellite images. For each layer k, a mask C^k denoting the area of clouds belonging to the associated layer is computed by EUMETSAT [Lutz99, Schmetz93]. Considering two successive images $\mathcal{I}(x,t)$ and $\mathcal{I}(x,t+1)$ defined on the 2D spatial domain Ω, the optical flow $v^k(x,t)$ of a layer k is then computed by minimizing the functional:

$$J(v^k)= \int_{C^k} \| \mathcal{I}(x,t+1)-\mathcal{I}(x,t)+ \nabla \mathcal{I}(x,t)\, v^k(x,t) \|^2 dx +\int_\Omega \|\mathrm{div}(v^k(x,t))\|^2 dx+ \int_\Omega \|\mathrm{curl}(v^k(x,t))\|^2 dx.$$

To assure the existence and the uniqueness of the solution, a spatial smoothing on the divergence and the vorticity of the unknown motion field is enforced [Corpetti02]. With this technique, a dense motion field $v^k_{obs}(x,t)$ is computed for each layer. However, as the obtained fields are frame to frame estimation, the successive results will suffer from time consistency.

Moreover, the masks of image data are sparse (the lower clouds are hidden by the upper ones in satellite images) and the estimation is noisy on the lower layers. Smoothing these velocity fields according to an appropriate dynamical law will denoise these estimations.

4. DYNAMICAL MODEL FOR ATMOSPHERIC LAYERS:

As atmosphere dynamics is governed by fluid flow laws, we may rely on Navier-Stokes equations in order to derive simplified dynamical models for short time propagation of layer mesoscale motion. As our aim is restricted to spatio-temporal smoothing at large scales, we rely instead on the filtered Navier-Stokes equations predicting wind fields at scales of order of 100 km, that is to say where rotational motion dominates divergent motion.

Let us denote v^k the density-weighted average horizontal wind related to the k-th layer. Denoting the vorticity by $\zeta^k = \mathrm{curl}\,(v^k)$ and the divergence by $D^k = \mathrm{div}\,(v^k)$ the dynamical system of a layer k may be expressed in its vorticity-divergence form [Heas07] as:

$$\begin{aligned} &\partial_t \zeta^k + v^k . \nabla \zeta^k + (\zeta^k + f^\Sigma) D^k = (C\Delta_x)^2 \ |\zeta^k|\ \Delta\zeta^k \\ &\partial_t D^k = (\Delta_x/6\sqrt{2})^2\ \Delta D^k \end{aligned} \qquad (1)$$

where C has a universal value of 0.17, Δ_x is the spatial grid, $f^{\Sigma} = 2\ \Lambda \sin(\Sigma)$ denotes the Coriolis coefficient, Σ are the viscous forces and Λ is the turbulent viscosity dissipation.
The curl and divergence completely determine the underlying 2D velocity field and the current velocity estimate can be recovered from these quantities up to a laminar flow. Indeed, the Helmholtz decomposition of the field into a sum of gradients of two potential functions is expressed as

$$v^k = \nabla^{\perp}\Psi + \nabla\Phi + v^k_{har},$$

where v^k_{har} is a harmonic transportation part ($\mathrm{div}(v^k_{har}) = \mathrm{curl}(v^k_{har}) = 0$) of the field v^k and where the stream function Ψ and the velocity potential Φ corresponds to the solenoidal and the irrotationnal part of the field. The latter are linked to divergence and vorticity through two Poisson equations. Expressing the solution of both equations as a convolution product with the 2D Green kernel G associated with the Laplacian operator: $\Psi = G * \zeta$, $\Phi = G * D$, the whole velocity field can be recovered with the equation:

$$v^k = \nabla^{\perp} (G * \zeta^k) + \nabla (G * D^k) + v^k_{har}, \qquad (2)$$

which can be efficiently solved in the Fourier domain. The harmonic transportation component v^k_{har} is recovered by substracting to the field v^k its solenoidal and irrotationnal parts.

5. FINAL SYSTEM:

We can now define all the components of the assimilation system allowing smoothing the velocities obtained from section 3 with the dynamic presented in section 4.
The final system enables the tracking of the vorticity and the divergence of the unknown vector fields v^k, referring to section 2 , we have $X^k = [\zeta^k, D^k]$.

The evolution model *M* is given by the mesoscale dynamics (1). Concerning the observations, we use the dense motion fields v^k_{obs} obtained with the technique described in section 3. From equation (2), the observation operator *H* is defined as:

$$H(X^k) = \nabla^{\perp} (G * \zeta^k) + \nabla (G * D^k) + v^k_{har}.$$

Hence the process minimizes:

$$J^k(\zeta_0^k, D_0^k) = \int_t \| v^k_{obs} - H(X^k(\zeta_0^k, D_0^k)) \|^2 dt,$$

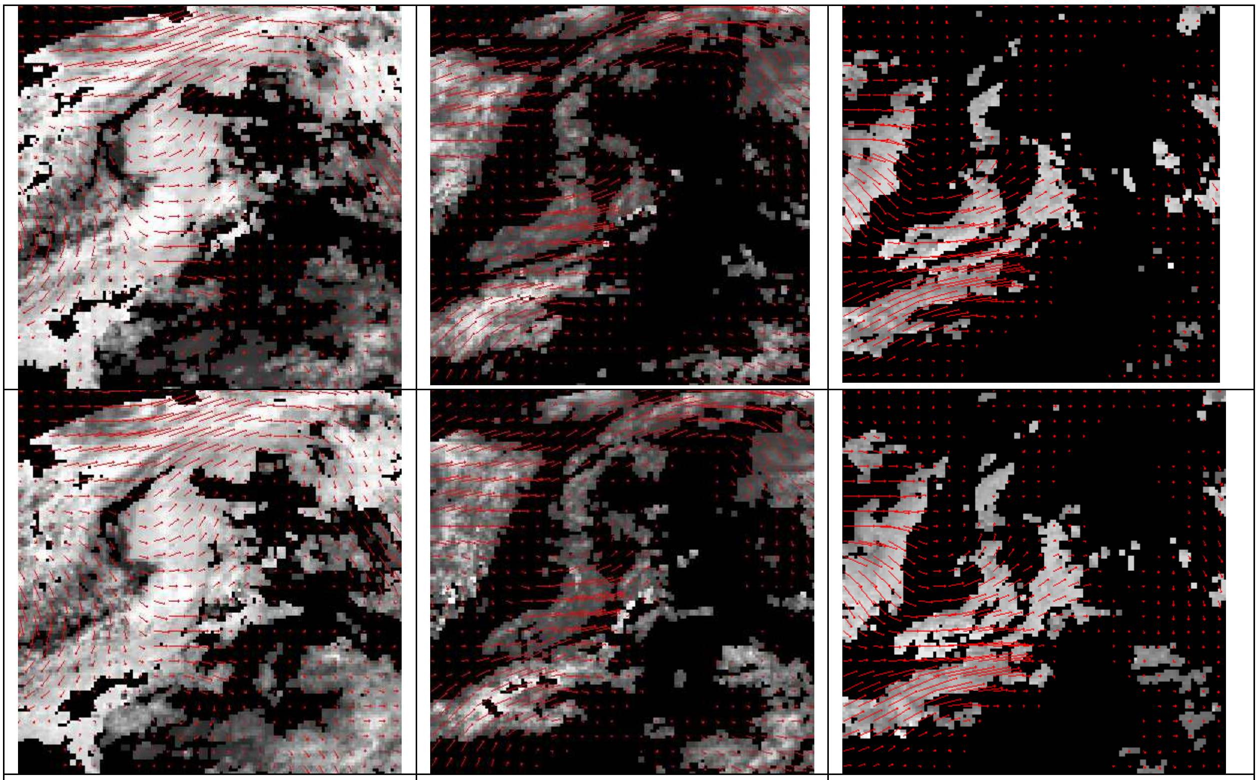
for each layer k with backward integrations of the adjoint model. More details on the construction of adjoint models can be found in [Talagrand87].

As the assimilation process is not insured to reach a global minimum, results depend on initialization. Thus, the initial conditions ζ_0^k and D_0^k are obtained, as the observations, with an optic-flow algorithm dedicated to atmospheric layers [Heas07].

RESULTS

We now turned to qualitative comparisons on a real meteorological image sequence.
The benchmark data was composed by a sequence of 10 METEOSAT Second Generation (MSG) images, showing top of cloud pressures with a corresponding cloud classification sequence. The 1024 x 1024 pixel images cover an area over the north Atlantic Ocean during part of one day (5-June-2004), at a rate of one image every 15 minutes. The spatial resolution is 3 kilometres at the center of the whole Earth image disk. Clouds from a cloud-classification provided by EUMETSAT were used to segment images into 3 broad layers, at low, intermediate and high altitude. In order to make the layering assumption valid, low resolution observations on an image grid of 128x128 pixels are obtained by smoothing and sub-sampling for each layer the original data.

By applying the methodology described in section 3 to the image at this coarser resolution, horizontal motion fields are estimated from the image sequence for these 3 layers. Estimated vector fields superimposed on pressure difference maps are displayed in figure 1 for each of the 3 layers. The motion fields estimated for the different layers on the cloudy observable parts is consistent with the visual inspection of the sequence. In particular, several motion differences between layers are very relevant. For instance, near the bottom left corner of the images, the lower layer possesses a southward motion while the intermediate layer moves northward. Moreover, the temporal coherence of the retrieved motion demonstrates the efficiency of this spatio-temporal smoothing under physical constraints.



Low layer | Middle layer | High layer

Figure 1: **First (above) and last (below) estimated horizontal wind fields superimposed on the cloud classification maps.**

Finally, we applied our technique on an Infra-red meteorological sequence showing Vince cyclone over north Atlantic. The sequence is composed by 20 satellite images acquired the 9 october 2005 from 00:00 up to 5:00 am (We thank the Laboratoire de Météorologie Dynamique for providing us this sequence).

Complete results in term of vorticity maps are presented on figure 2. We only consider one layer in this application. Column b exhibits the different vorticity maps of the initial motion fields used as noisy measurements. These motion observations present temporal inconsistencies and discontinuities, whereas the recovered vorticity maps shown on column c are more compliant with the vorticity conservation law. A set of reconstructed motion fields is given in figure 3.

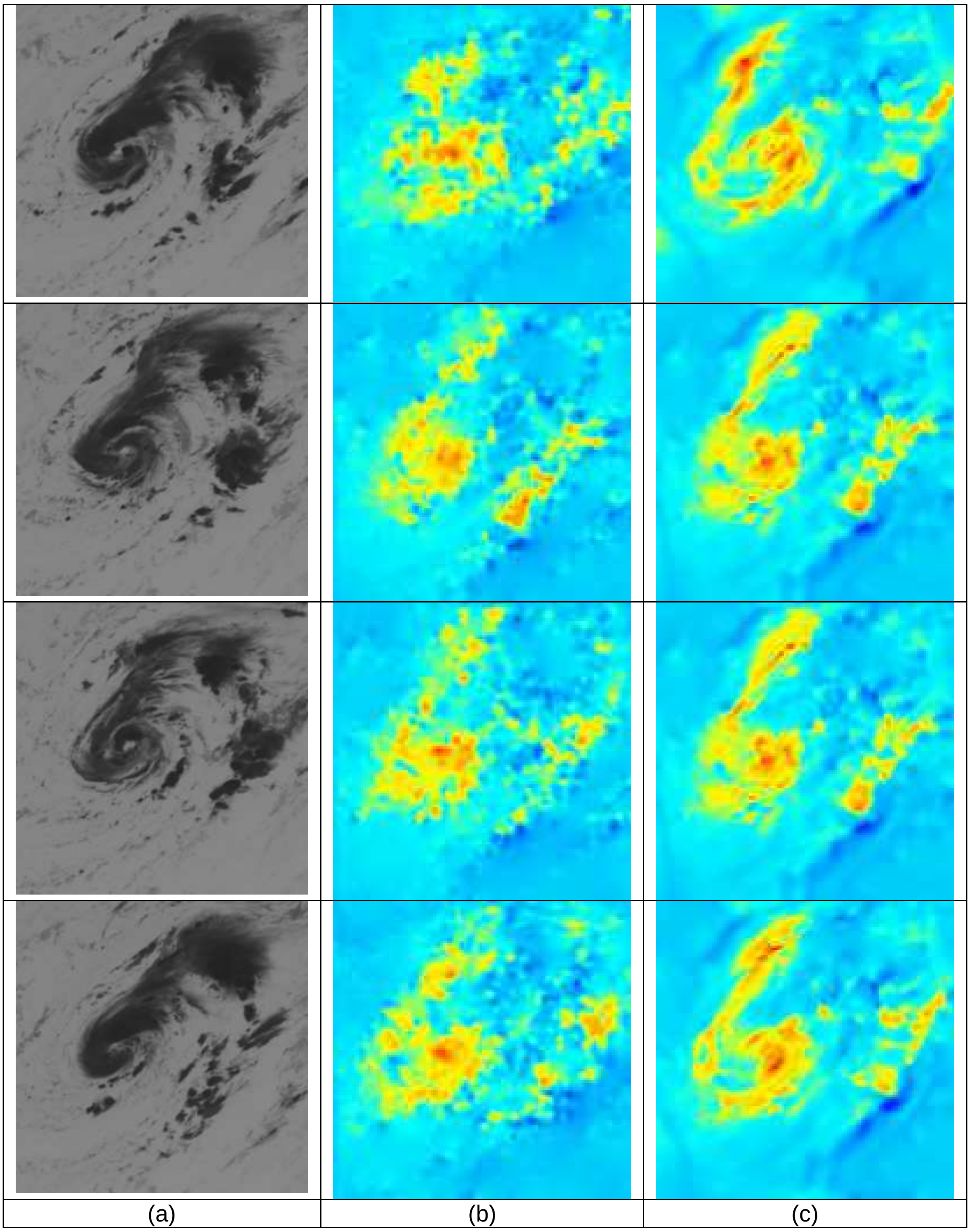
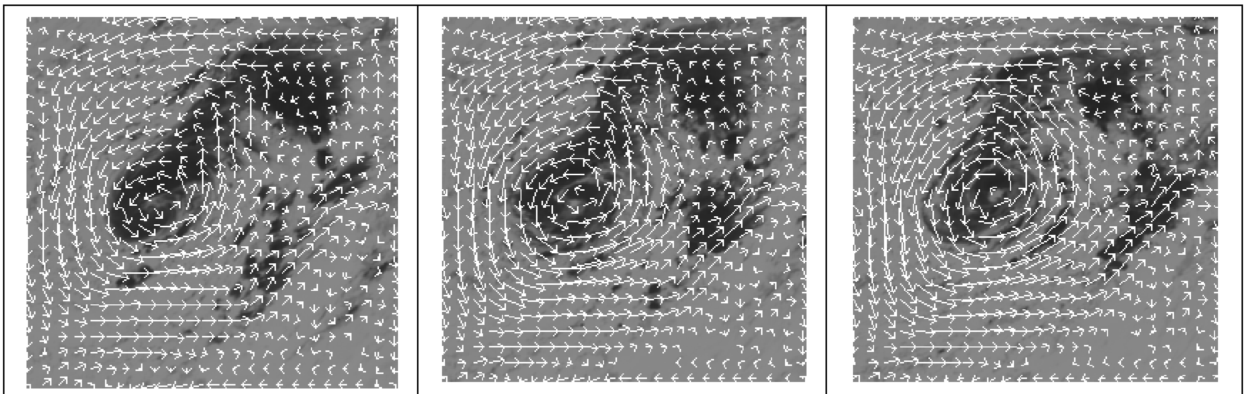


Figure 2: **Cyclone sequence: (a) Cyclone sequence. (b) Sample of observed vorticity maps. (c) Vorticity maps corresponding to the recovered motion fields.**



T=1 T=10 T=20

Figure 3: **Cyclone sequence: Three reconstructed motion fields.**

CONCLUSIONS

In this paper, we have presented a new method for estimating time-consistent horizontal winds in a stratified atmosphere from satellite image sequences.

The proposed estimator applies on a set of sparse image observations related to a multi-layer atmosphere, which verify independent atmospheric models.
In order to manage the incomplete and noisy observations while considering this non-linear physical model, a variational assimilation scheme is proposed. This process estimates time-consistent motion fields related to the layer components.

In view of the various meteorological studies relying on the analysis of experimental data of atmospheric dynamics, we believe that the proposed multi-layer horizontal wind field estimation technique constitutes a valuable tool.

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